

NAG Toolbox for MATLAB

c02ak

1 Purpose

c02ak determines the roots of a cubic equation with real coefficients.

2 Syntax

```
[zeror, zeroi, errest, ifail] = c02ak(u, r, s, t)
```

3 Description

c02ak attempts to find the roots of the cubic equation

$$uz^3 + rz^2 + sz + t = 0,$$

where u, r, s and t are real coefficients with $u \neq 0$. The roots are located by finding the eigenvalues of the associated 3 by 3 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} 0 & 0 & -t/u \\ 1 & 0 & -s/u \\ 0 & 1 & -r/u \end{pmatrix}.$$

The eigenvalues are obtained by a call to f08pe. Further details can be found in Section 8.

To obtain the roots of a quartic equation, c02al can be used.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **u – double scalar**

u , the coefficient of z^3 .

Constraint: $u \neq 0.0$.

2: **r – double scalar**

r , the coefficient of z^2 .

3: **s – double scalar**

s , the coefficient of z .

4: **t – double scalar**

t , the constant coefficient.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **zeror(3)** – double array

2: **zeroi(3)** – double array

zeror(*i*) and **zeroi**(*i*) contain the real and imaginary parts, respectively, of the *i*th root.

3: **errest(3)** – double array

errest(*i*) contains an approximate error estimate for the *i*th root.

4: **ifail** – int32 scalar

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, **u** = 0.0.

ifail = 2

The companion matrix *H* cannot be formed without overflow.

ifail = 3

The iterative procedure used to determine the eigenvalues has failed to converge.

7 Accuracy

If **ifail** = 0 on exit, then the *i*th computed root should have approximately $|\log_{10}(\mathbf{errest}(i))|$ correct significant digits.

8 Further Comments

The method used by the function consists of the following steps, which are performed by routines from LAPACK in Chapter F08.

(a) Form *H*.

(b) Apply a diagonal similarity transformation to *H* (to give *H'*).

(c) Calculate the eigenvalues and Schur factorization of *H'*.

(d) Calculate the left and right eigenvectors of *H'*.

(e) Estimate reciprocal condition numbers for all the eigenvalues of *H'*.

(f) Calculate approximate error estimates for all the eigenvalues of *H'* (using the 1-norm).

9 Example

```
u = 1;  
r = 3;  
s = 9;
```

```
t = -13;  
[zeror, zeroi, errest, ifail] = c02ak(u, r, s, t)
```

```
zeror =  
    1.0000  
   -2.0000  
   -2.0000  
zeroi =  
     0  
     3  
    -3  
errest =  
    1.0e-14 *  
    0.1006  
    0.1041  
    0.1041  
ifail =  
        0
```
